1 > 3 Species with Variable Parms: PROOFs;

2. > $restart : with(plots) : with(StringTools) :$
$with(DEtools) : with(ColorTools) : with(plots) : with(geom3d) : with(plottools) :$
$with(FileTools) : with(Optimization) : with(LinearAlgebra) :$
$with(VectorCalculus) :$

3. > $FormatTime($"%I:%M-%p----%d-%b-%Y"$); currentdir(\,); printlevel := 1 :$

"10:55-AM----01-Jun-2020"

"C:\Users\nn\Documents\2 research\2017 summer non const parms\current" **(1)**

4. > *##TOP##*

1. Proof Theorem 1
2. Proof Theorem 2
3. Corollary
4. Solutions
5. Fig 3 What to Parms mean?
6. Fig 4 parms set 1
7. 3Dto2D
8. Fig 10
9. Proof Theorem 4

5. > *#Proofs of theorems 1& 2*

6. > *####### constants parms to graph 2d geometry*

7. > $eqV2D := -v1 + v2 \cdot U \cdot \left(1 - \frac{V}{kv}\right) = 0$

8. > $eqU2D := -u1 + u2 \cdot V \left(1 - \frac{U}{ku}\right) = 0$

$$eqV2D := -v1 + v2\, U \left(1 - \frac{V}{kv}\right) = 0$$

$$eqU2D := -u1 + u2\, V \left(1 - \frac{U}{ku}\right) = 0 \quad \textbf{(2)}$$

9. > $V1 := solve(eqV2D, V)$

10. > $limVinf := limit(V1, U = \text{infinity})$

11. > $limVv := limit\left(V1, U = \frac{v1}{v2}\right)$

12. > $limV0 := limit(V1, U = 0, right)$

$$V1 := \frac{(U\, v2 - v1)\, kv}{v2\, U}$$

$$limVinf := kv$$

$$limVv := 0$$

$$limV0 := -\text{signum}\left(\frac{v1\, kv}{v2}\right) \infty \quad \textbf{(3)}$$

13. > $U1 := solve(eqU2D, U)$

14. > $limUinf := limit(U1, V = \text{infinity})$

15. > $limUu := limit\left(U1, V = \frac{u1}{u2}\right)$

16. > $limU0 := (limit(U1, V = 0, right))$

$$U1 := \frac{(V\,u2 - u1)\,ku}{u2\,V}$$
$$limUinf := ku$$
$$limUu := 0$$
$$limU0 := -\mathrm{signum}\left(\frac{u1\,ku}{u2}\right)\infty \qquad (4)$$

17. > ## *Theorem 1* ####################
18. > ## *solve for Vstar and Ustar*
19. > $eqV3D := -v1 + v2 \cdot U \cdot \left(1 - \frac{V}{kv}\right) - v3\,A = 0$
20. > $eqU3D := -u1 + u2 \cdot V \cdot \left(1 - \frac{U}{ku}\right) = 0$
21. > $eqA3D := -a1 + a2 \cdot V = 0$
22. > $Vs3D := solve(eqA3D, V)$
23. > $Us3D := solve(subs(V = Vs3D, eqU3D), U)$

$$eqV3D := -v1 + v2\,U\left(1 - \frac{V}{kv}\right) - v3\,A = 0$$
$$eqU3D := -u1 + u2\,V\left(1 - \frac{U}{ku}\right) = 0$$
$$eqA3D := V\,a2 - a1 = 0$$
$$Vs3D := \frac{a1}{a2}$$
$$Us3D := \frac{(a1\,u2 - a2\,u1)\,ku}{u2\,a1} \qquad (5)$$

24. > ## *Theorem 2* ############
25. > #*To Showm*
26. > $Astar := \frac{ku \cdot v2}{kv \cdot v3}\left(kv - \frac{a2 \cdot u1}{a1 \cdot u2} kv - \frac{v1 \cdot kv}{v2 \cdot ku} - \frac{a1}{a2} + \frac{u1}{u2}\right)$
27. > ##*solve the equations for Astar,. call it As1; then simplify*

$$Astar := \frac{ku\,v2\left(kv - \frac{u1\,a2\,kv}{u2\,a1} - \frac{v1\,kv}{ku\,v2} - \frac{a1}{a2} + \frac{u1}{u2}\right)}{kv\,v3} \qquad (6)$$

28. > $As1 := solve(eqV3D, A)$;
29. > $As2 := simplify(subs([U = Us3D, V = Vs3D], As1))$
30. > $simplify(simplify(As2 - Astar))$

$$As1 := -\frac{U\,V\,v2 - U\,kv\,v2 + kv\,v1}{kv\,v3}$$
$$As2 := \frac{-a1^2\,ku\,u2\,v2 + (v2\,(kv\,u2 + u1)\,ku - kv\,u2\,v1)\,a2\,a1 - a2^2\,ku\,kv\,u1\,v2}{u2\,a2\,a1\,kv\,v3}$$
$$0 \qquad (7)$$

31. > #### *Corollary to Theorem 2* #####
32. > $eqV2D$
33. > $simplify(subs([U = Us3D, V = Vs3D], lhs(eqV2D)))$
34. > $simplify(\% - v3 \cdot As2)$

$$-v1 + v2\, U\left(1 - \frac{V}{kv}\right) = 0$$

$$\frac{-a1^2\, ku\, u2\, v2 + (v2\,(kv\, u2 + u1)\, ku - kv\, u2\, v1)\, a2\, a1 - a2^2\, ku\, kv\, u1\, v2}{a1\, a2\, kv\, u2}$$

$$0 \qquad \mathbf{(8)}$$

35. > ## ***Marginal analysis of theorems 1 & 2***

36. > ##TS: $DA = \frac{kv}{ku} DU - DV$

37. > $Us3D;\ Vstar := \frac{a1}{a2};\ A3s;$

38. > $DV := diff(Vstar, a2)$

39. > $DU := diff(Us3D, a2)$

40. > $DA := collect(simplify(diff(As3, a2)), u2)$

41. > $\left(\frac{kv}{ku} DU - DV - DA\right)$

42. > $eqA := DA = 0$

43. > $a2star := (solve(eqA, a2))[1];$

44. >

45. > $diff(DV, a2)$

46. > $\frac{a1}{a2star}$

$$\frac{(a1\, u2 - a2\, u1)\, ku}{u2\, a1}$$

$$Vstar := \frac{a1}{a2}$$

$$A3s$$

$$DV := -\frac{a1}{a2^2}$$

$$DU := -\frac{u1\, ku}{u2\, a1}$$

$$DA := 0$$

$$-\frac{kv\, u1}{u2\, a1} + \frac{a1}{a2^2}$$

$$eqA := 0 = 0$$

$$a2star := a2_1$$

$$\frac{2\, a1}{a2^3}$$

$$\frac{a1}{a2_1} \qquad \mathbf{(9)}$$

47. > ## ***Solutions***

48. > ##2 D problem variable parms

49. > $unassign('ku','kv','v1','v2','v3','u1','u2','a1','a2','s')$

50. > $Vdot2D := s \rightarrow -v1 + v2 \cdot U \cdot \left(1 - \frac{V}{kv}\right)$:; $Vdot2D(s)$;

51. > $Udot2D := s \rightarrow -u1(s) + u2(s) \cdot V \left(1 - \frac{U}{ku}\right)$: $Udot2D(s)$

52. > ### 2D equations ##########

53. > $eqV2D := s \rightarrow Vdot2D(s) = 0$;

54. > $eqU2D := s \rightarrow Udot2D(s) = 0$;

55. > $UVsol := s \rightarrow solve([eqV2D(s), eqU2D(s)], [U, V])$;

$$-v1 + v2\, U \left(1 - \frac{V}{kv}\right)$$

$$-u1(s) + u2(s)\, V \left(1 - \frac{U}{ku}\right)$$

$$eqV2D := s \rightarrow Vdot2D(s) = 0$$

$$eqU2D := s \rightarrow Udot2D(s) = 0$$

$$UVsol := s \rightarrow solve([eqV2D(s), eqU2D(s)], [U, V]) \tag{10}$$

56. > #3 D problem variable parms analytic solution

57. > $Vdot3D := s \rightarrow -v1 + v2 \cdot U \cdot \left(1 - \frac{V}{kv}\right) - v3\, A$:

58. > $Udot3D := s \rightarrow -u1(s) + u2(s) \cdot V \cdot \left(1 - \frac{U}{ku}\right)$:

59. > $Adot3D := s \rightarrow -a1(s) + a2(s) \cdot V$:

60. > $Vs3D := s \rightarrow solve(Adot3D(s) = 0, V)$: $Vs3D(s)$

61. > $Us3D := s \rightarrow (solve(subs(V = Vs3D(s), Udot3D(s)), U))$: $Us3D(s)$

62. > $As1 := s \rightarrow solve(Vdot3D(s), A)$:

63. > $As2 := s \rightarrow subs(U = Us3D(s), As1(s))$:

64. > $As3D := s \rightarrow subs(V = Vs3D(s), As2(s))$:; $simplify(As3D(s))$

$$\frac{a1(s)}{a2(s)}$$

$$-\frac{ku\,(u1(s)\, a2(s) - u2(s)\, a1(s))}{u2(s)\, a1(s)}$$

$$\frac{1}{u2(s)\, a2(s)\, a1(s)\, kv\, v3} \left(-u2(s)\, a1(s)^2\, ku\, v2 + a2(s)\, (kv\,(ku\, v2 - v1)\, u2(s) + ku\, v2\, u1(s))\, a1(s) - u1(s)\, a2(s)^2\, ku\, kv\, v2\right) \tag{11}$$

65. > $Vstar := s \rightarrow Vs3D(s)$; $Ustar := s \rightarrow Us3D(s)$; $Astar := s \rightarrow As3D(s)$

$$Vstar := s \rightarrow Vs3D(s)$$

$$Ustar := s \rightarrow Us3D(s)$$

$$Astar := s \rightarrow As3D(s) \tag{12}$$

> $expand\left(\frac{ku\,(u1\ a2 + u2)}{u2\, a1}\right)$

$$\frac{ku\, u1\, a2}{u2\, a1} + \frac{ku}{a1} \tag{13}$$

66. > ############### GO TO TOP ###############

67. > #Define parms dependent on S logistic

68. > unassign('s','La1','ma1','ca','ma2','La2','da','Lu1','cu','Lu2','du')

69. > $a1 := s \to \frac{La1}{1+e^{-ma1\,(s-ca1)}}$:

70. > $a2 := s \to La2 - \frac{La2}{1+e^{-ma2\,(s-ca2)}}$:

71. > $u1 := s \to \frac{Lu1}{1+e^{-mu1\,(s-cu1)}}$:

72. > $u2 := s \to Lu2 - \frac{Lu2}{1+e^{-mu2\,(s-cu2)}}$:

73. > ##First Derivative of Vstar### part 1#######EQ11

74. > $collect\left(simplify\left(\frac{d}{ds}\,Vstar(s)\right), s\right)$

75. > # simplify ((ma1+ma2)*exp(ma2*(s-ca+da)-ma1*(s-ca))+ma1*exp(ma1*(-s+ca))+ma2*exp(ma2*(s-ca+da)))*La1

$$\frac{\left((ma1+ma2)\,e^{ma1\,(-s+ca1)\,+\,ma2\,(s-ca2)}+e^{ma1\,(-s+ca1)}\,ma1+ma2\,e^{ma2\,(s-ca2)}\right)La1}{La2\,\left(1+e^{ma1\,(-s+ca1)}\right)^2} \qquad (14)$$

76. > ##First Derivative of Vstar## part 2###

77. > ######EQ12##################

78. > $\frac{\partial}{\partial s}\,\frac{A1(s)}{A2(s)}$; $unassign(A1(s), A2(s))$

$$\frac{\frac{d}{ds}\,A1(s)}{A2(s)}-\frac{A1(s)\left(\frac{d}{ds}\,A2(s)\right)}{A2(s)^2} \qquad (15)$$

79. > ############### GO TO TOP ###############

80. > ###FIG 3 ####What do parms mean?

81. > $mu1 := .5;\ : demom1 := t \to \frac{0.1}{1+e^{-mu1\,(t-5)}}$

82. > $mu2 := 1;\ demom2 := t \to \frac{0.1}{1+e^{-mu2\,(t-5)}}$

83. > $cu1 := 10;\ democ1 := t \to \frac{0.1}{1+e^{-mu1\,(t-cu1)}}$:

84. > $cu2 := 12;\ democ2 := t \to \frac{0.1}{1+e^{-mu1\,(t-cu2)}}$:

85. > *unassign*('s') : *plot*([*demom1*(*s*), *demom2*(*s*), *democ1*(*s*), *democ2*(*s*)], *s*=-2 ..20, *thickness* = [3, 1, 1, 1], *legend* = ["m1", "m2>m1", "c1", "c2>c1"], *title* = "Impact of Parameters on Logistic Equation", *linestyle* = [*dash*, *dashdot*, *solid*, *dot*], *legendstyle* = [*font* = [*roman*, 20]], *titlefont* = [*roman*, 25], *labelfont* = [*roman*, 25])

$$\mu 1 := 0.5$$

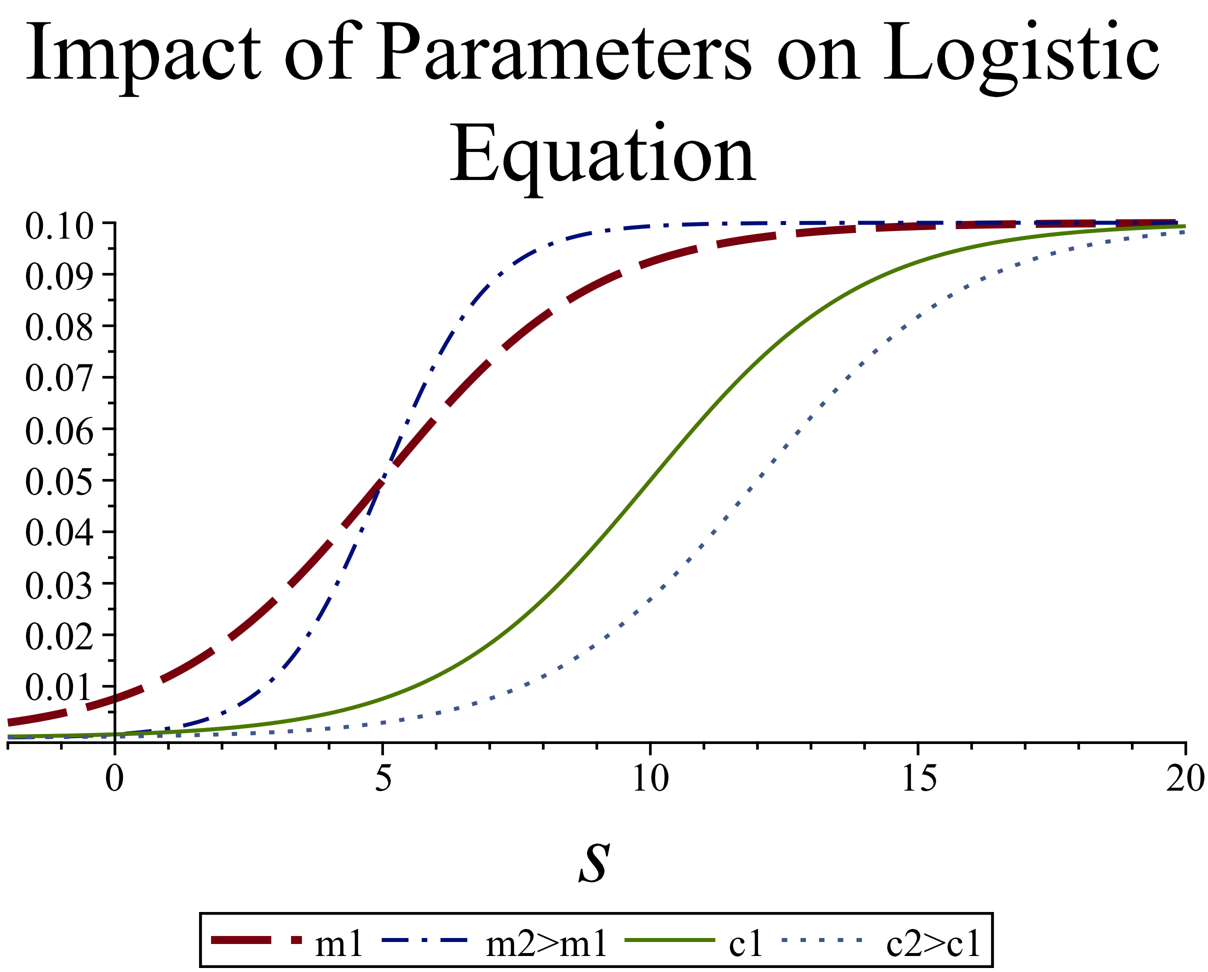
$$demom1 := t \rightarrow 0.1 \cdot 1 \frac{1}{1 + e^{VectorCalculus:-`-`(\mu 1\,(t + (-5)))}}$$

$$\mu 2 := 1$$

$$demom2 := t \rightarrow 0.1 \cdot 1 \frac{1}{1 + e^{VectorCalculus:-`-`(\mu 2\,(t + (-5)))}}$$

$$cu1 := 10$$

$$cu2 := 12$$



86. > ################ GO TO TOP ################

87. > ########3D system

88. > **unassign('s') :**

89. > $e1 := \frac{d}{dt} U(t) = -u1(s) \cdot U(t) + u2(s) \cdot V(t) \cdot U(t) \cdot \left(1 - \frac{U(t)}{ku}\right)$

90. > $e2 := \frac{d}{dt} V(t) = -v1 \cdot V(t) + v2 \cdot V(t) \cdot U(t) \cdot \left(1 - \frac{V(t)}{kv}\right) - v3 \cdot A(t) \cdot V(t)$

91. > $e2b := \frac{d}{dt} V(t) = -v1 \cdot V(t) + v2 \cdot V(t) \cdot U(t) \cdot \left(1 - \frac{V(t)}{kv}\right)$

92. > $e3 := \frac{d}{dt} A(t) = -a1(s) \cdot A(t) + a2(s) \cdot V(t) \cdot A(t)$

$$e1 := \frac{d}{dt}\,U(t) = -\frac{Lu1\,U(t)}{1+e^{-0.5\,s+5.0}} + \left(Lu2 - \frac{Lu2}{1+e^{-s+12}}\right) V(t)\,U(t)\left(1-\frac{U(t)}{ku}\right)$$

$$e2 := \frac{d}{dt}\,V(t) = -v1\,V(t) + v2\,V(t)\,U(t)\left(1-\frac{V(t)}{kv}\right) - v3\,A(t)\,V(t)$$

$$e2b := \frac{d}{dt}\,V(t) = -v1\,V(t) + v2\,V(t)\,U(t)\left(1-\frac{V(t)}{kv}\right)$$

$$e3 := \frac{d}{dt}\,A(t) = -\frac{La1\,A(t)}{1+e^{-ma1\,(s-ca1)}} + \left(La2 - \frac{La2}{1+e^{-ma2\,(s-ca2)}}\right) V(t)\,A(t) \quad \textbf{(16)}$$

93. > $autonomous(\{e1, e2, e3\}, \{U(t), V(t), A(t)\}, t);$

true **(17)**

94. > # *find null clines*

95. > $unassign('s'); ncu := s \to 0 = -u1(s)\cdot U + u2(s)\cdot V\cdot U\cdot\left(1-\frac{U}{ku}\right);$

96. > $ncv := s \to 0 = -v1\cdot V + v2\cdot U\cdot V\cdot\left(1-\frac{V}{kv}\right) - v3\cdot A\cdot V$

97. > $nca := s \to 0 = -a1(s)\cdot A + a2(s)\cdot V\cdot A$

$$ncu := s \to 0 = VectorCalculus:\text{-}`\text{-}`(u1(s)\;U) + u2(s)\;V\,U\left(1 + VectorCalculus:\text{-}`\text{-}`\left(U \cdot 1\,\frac{1}{ku}\right)\right)$$

$$ncv := s \to 0 = VectorCalculus:\text{-}`\text{-}`(v1\;V) + v2\;U\,V\left(1 + VectorCalculus:\text{-}`\text{-}`\left(V\cdot 1\,\frac{1}{kv}\right)\right) + VectorCalculus:\text{-}`\text{-}`(v3\;A\;V)$$

$$nca := s \to 0 = VectorCalculus:\text{-}`\text{-}`(a1(s)\;A) + a2(s)\;V\,A \quad \textbf{(18)}$$

98. > ############### GO TO TOP ###############

99. > ## *PROOF of THEOREM 4*

100. > ##*TOP*

101. > $unassign('kv','ku','u1','u2','v1','v2','v3')$

102. > FormatTime("%I:%M-%p---%d-%b-%Y")

103. > ## *NON NORMALIZED variables CONSTANT limits*

104. > ###################### *the differential equations*

105. > $Vdot := -v1\cdot V + v2\cdot U\cdot V\cdot\left(1-\frac{V}{kv}\right)$

106. > $Udot := -u1\cdot U + u2\cdot U\cdot V\cdot\left(1-\frac{U}{ku}\right)$

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$$Vdot := -v1\;V + v2\;V\,U\left(1-\frac{V}{kv}\right)$$

$$Udot := -u1\;U + u2\;U\,V\left(1-\frac{U}{ku}\right) \quad \textbf{(19)}$$

107. > ################## *the jacobian* ######

108. > $J := simplify(expand(Jacobian([Vdot, Udot], [V, U])));$ ############ *EQ 19*

109. > $TJ := Trace(J)$; ############### *EQ 21*

110. > $DJ := Determinant(J)$; ######################## EQ 22

$$J := \begin{bmatrix} \frac{(U\,v2 - v1)\,kv - 2\,v2\,U\,V}{kv} & -\frac{v2\,V\,(-kv + V)}{kv} \\ -\frac{u2\,U\,(-ku + U)}{ku} & \frac{ku\,(V\,u2 - u1) - 2\,u2\,U\,V}{ku} \end{bmatrix}$$

$$TJ := \frac{(U\,v2 - v1)\,kv - 2\,v2\,U\,V}{kv} + \frac{ku\,(V\,u2 - u1) - 2\,u2\,U\,V}{ku}$$

$$DJ := \frac{1}{kv\,ku}\left(3\,U^2\,V^2\,u2\,v2 - U^2\,V\,kv\,u2\,v2 - U\,V^2\,ku\,u2\,v2 + 2\,U\,V\,ku\,u1\,v2 + 2\,U\,V\,kv\,u2\,v1 - U\,ku\,kv\,u1\,v2 - V\,ku\,kv\,u2\,v1 + ku\,kv\,u1\,v1\right) \quad (20)$$

111. > ################ at the origin DJ >0#######

112. > $DJat0 := simplify(subs(V=0, subs(U=0, DJ)))$

113. > ########t the origin, TJ<0

114. > $TJat0 := subs(V=0, subs(U=0, TJ))$

115. > ## at the origin, D>0, T<0 therefore stable

$$DJat0 := u1\,v1$$

$$TJat0 := -u1 - v1 \quad (21)$$

116. > ################nc: nullclines#######################

117. > $e1 := Vdot = 0$

118. > $e2 := Udot = 0$

$$e1 := -v1\,V + v2\,V\,U\left(1 - \frac{V}{kv}\right) = 0$$

$$e2 := -u1\,U + u2\,U\,V\left(1 - \frac{U}{ku}\right) = 0 \quad (22)$$

119. > #simply by dividing eq1 by V and eq2 by U

120. > $eq1 := sort\left(-expand\left(\frac{kv}{v2} \cdot simplify\left(\frac{e1}{V}\right)\right), [U, V]\right)$

121. > $eq2 := sort\left(-expand\left(\frac{ku}{u2} \cdot simplify\left(\frac{e2}{U}\right)\right), [U, V]\right)$

122. > ## these do not apply at the origin since we divided by U and V

$$eq1 := U\,V - kv\,U + \frac{v1\,kv}{v2} = 0$$

$$eq2 := U\,V - ku\,V + \frac{u1\,ku}{u2} = 0 \quad (23)$$

123. > ####solve Equations for Nullclines as functions of U inthe UV space ###############

124. > $Vnc1 := solve(eq1, V)$; $limit(Vnc1, U = \text{infinity})$
#equation for NC1, limit shows it is concave

$$Vnc1 := \frac{(U\,v2 - v1)\,kv}{v2\,U}$$

$$kv \quad (24)$$

125. > $Vnc2 := solve(eq2, V)$; $limit(Vnc2, U = ku, left)$

#equation for NC2, limit shows it is convex

$$Vnc2 := -\frac{u1\,ku}{u2\,(-ku+U)}$$

$$\operatorname{signum}\left(\frac{u1\,ku}{u2}\right)\,\infty \qquad \textbf{(25)}$$

126. > *#### evaluate the jacobian at the Critical Points ##################*

127. > *J*

$$\begin{bmatrix} \frac{(U\,v2-v1)\,kv-2\,v2\,U\,V}{kv} & -\frac{v2\,V\,(-kv+V)}{kv} \\ -\frac{u2\,U\,(-ku+U)}{ku} & \frac{ku\,(V\,u2-u1)-2\,u2\,U\,V}{ku} \end{bmatrix} \qquad \textbf{(26)}$$

128. > *#simplify by adding 0 to two terms. Note this applies to critical points NOT at the origin*

129. > $j11 := simplify\left(J[1,1]+\frac{v2}{kv}\cdot lhs(eq1)\right)$

130. > $j22 := simplify\left(J[2,2]+\frac{u2\cdot lhs(eq2)}{ku}\right)$

$$j11 := -\frac{v2\,U\,V}{kv}$$

$$j22 := -\frac{u2\,U\,V}{ku} \qquad \textbf{(27)}$$

131. > *# ##################Jacobian at the non origin CPs*

132. > $Jcp := J:$

133. > $Jcp[1,1] := j11 : Jcp[2,2] := j22 :$

134. > $Jcp := simplify(Jcp)$

$$Jcp := \begin{bmatrix} -\frac{v2\,U\,V}{kv} & -\frac{v2\,V\,(-kv+V)}{kv} \\ -\frac{u2\,U\,(-ku+U)}{ku} & -\frac{u2\,U\,V}{ku} \end{bmatrix} \qquad \textbf{(28)}$$

135. > $Tcp := factor(Trace(Jcp))$

136. > *## Tcp will be negative* ***at the CPs, ie they are sinks*** **or *saddle points*** *except at the origin #######*

$$Tcp := -\frac{U\,V\,(ku\,v2+kv\,u2)}{kv\,ku} \qquad \textbf{(29)}$$

137. > $DetJcp := Determinant(Jcp)$

138. > *## this will be positive (ie stable) for larger U,V and negative (SP) for small values*

139. > *# while this confirms the result, it does not prove that the farther cp will be far enough away from the origin to be stable.*

140. > *## it would be sufficient to show that this is positive for U>UD2*

141. > *#since we are only concerned about the sign, we can simplify*

142. > $DetJcps := \frac{DetJcp\cdot ku\cdot kv}{u2\cdot v2\cdot U\cdot V}$

143. > *## Note that DetJcp is increasing in U*

$$DetJcp := \frac{v2\ U\ V\ u2\ (U\ kv + V\ ku - ku\ kv)}{kv\ ku}$$

$$DetJcps := U\ kv + V\ ku - ku\ kv \qquad \textbf{(30)}$$

144. > *unassign('ku','kv','v1','v2','u1','u2')*

145. > *#find the line where the determinant of the jacobian is 0*

146. > $Vdet := factor(solve(DetJcps = 0, V))$; $VDet := U \to Vd : VDet(U)$

$$Vdet := -\frac{kv\ (-ku + U)}{ku}$$

$$Vd \qquad \textbf{(31)}$$

147. > *Vnc1*

$$\frac{(U\ v2 - v1)\ kv}{v2\ U} \qquad \textbf{(32)}$$

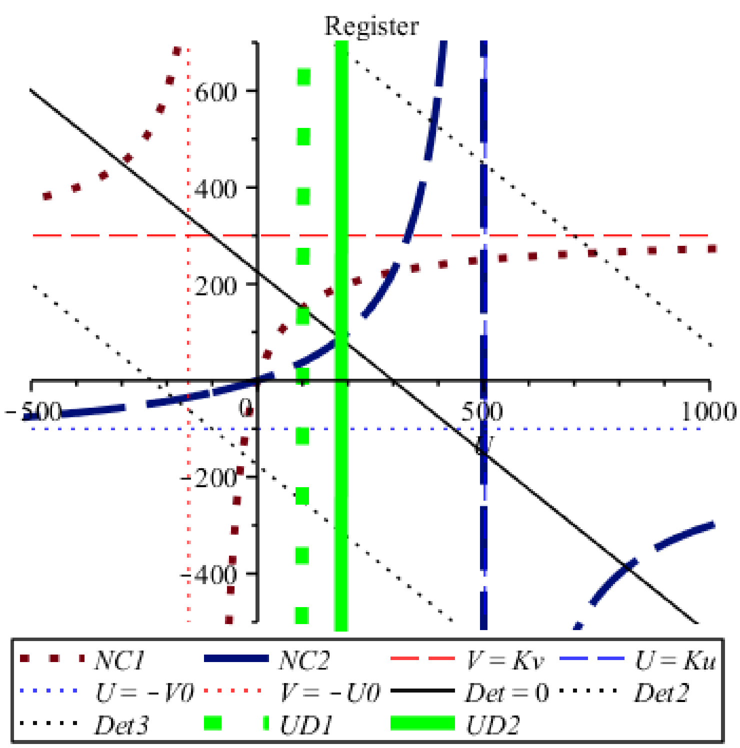
148. > $UDet1 := solve(Vnc1 = Vdet, U)$;
 # value of V where nullcline NC1 intersects Det=0

149. > $UDet2 := solve(Vnc2 = Vdet, U)$;
 # value of V where nullcline NC2 intersects Det=0

$$UDet1 := \frac{\sqrt{v2\ ku\ v1}}{v2},\ -\frac{\sqrt{v2\ ku\ v1}}{v2}$$

$$UDet2 := \frac{\left(kv\ u2 + \sqrt{kv\ u1\ u2}\right)\ ku}{kv\ u2},\ -\frac{\left(-kv\ u2 + \sqrt{kv\ u1\ u2}\right)\ ku}{kv\ u2} \qquad \textbf{(33)}$$

150. > ***###Figure 11 #############***



151. > *#it will be sufficient to show that UD2 is greater than UD1.*

152. > *########Intersections of Null Clines and Det=0 ##################*

153. > *####note thatach of these has two solutions.*

154. > *# so for NC1 we need the larger value for U in the intersection with Det=0 and for NC2 we need the smaller value for U. So for NC1 (U1) we take the term with the radical positive and for NC2 (U2) we take the term with the radical negative. Now we need to show that U2>U1.*

155. > unassign('ku','kv','u0','v0','u1','u2','v1','v2');

156. > $U1 := expand(UDet1[1]);$

157. > $U2 := expand(UDet2[2]);:$

$$U1 := \frac{\sqrt{v2\,ku\,v1}}{v2}$$

$$U2 := ku - \frac{ku\sqrt{kv\,u1\,u2}}{kv\,u2} \quad \textbf{(34)}$$

158. > *###when is U2 >U1?*

159. > unassign('ku','kv','u0','v0','u1','u2','v1','v2','v3')

160. > $\text{delta} := expand(kv \cdot u2 \cdot v2 \cdot simplify(U2 - U1))$

$$\delta := ku\,kv\,u2\,v2 - ku\sqrt{kv\,u1\,u2}\;v2 - \sqrt{v2\,ku\,v1}\;kv\,u2 \quad \textbf{(35)}$$

161. > *##When is delta >0 ?*

162. > nops(delta)

163. > $d1 := op(1, \text{delta})$

164. > $d2 := op(2, \text{delta})$

165. > $d3 := op(3, \text{delta})$

$$3$$

$$d1 := ku\,kv\,u2\,v2$$

$$d2 := -ku\sqrt{kv\,u1\,u2}\;v2$$

$$d3 := -\sqrt{v2\,ku\,v1}\;kv\,u2 \quad \textbf{(36)}$$

166. > $f2 := factor\left(\frac{d1}{2} + d2\right);$

167. > $f3 := factor\left(\frac{d1}{2} + d3\right);$

168. > $simplify(f2 + f3 - \text{delta})$

$$f2 := -\frac{1}{2}\,ku\,v2\,\left(-kv\,u2 + 2\sqrt{kv\,u1\,u2}\right)$$

$$f3 := -\frac{1}{2}\,kv\,u2\,\left(-ku\,v2 + 2\sqrt{v2\,ku\,v1}\right)$$

$$0 \quad \textbf{(37)}$$

169. > $e20 := \left(expand\left(\frac{f2}{ku \cdot kv \cdot v2}\right)\right) > 0;$

$$e20 := 0 < \frac{1}{2}\,u2 - \frac{\sqrt{kv\,u1\,u2}}{kv} \quad \textbf{(38)}$$

170. > $e21 := -\mathrm{sqrt}(kv) simplify(op(2, rhs(e20)), symbolic) < op(1, rhs(e20)) \cdot \mathrm{sqrt}(kv)$

$$e21 := \sqrt{u1}\,\sqrt{u2} < \frac{1}{2}\,u2\sqrt{kv} \quad \textbf{(39)}$$

171. > $e22 := lhs(e21)^2 < rhs(e21)^2$

(40)

$$e22 := u1\,u2 < \frac{1}{4}\,u2^2\,kv \quad (40)$$

172. > *solve*(*e22*, *kv*) assuming (*u2* > 0)

$$\left\{\frac{4\,u1}{u2} < kv\right\} \quad (41)$$

173. > ###\
#########

174. > $e30 := \left(expand\left(\frac{f3}{ku \cdot kv \cdot u2}\right)\right) > 0;$

$$e30 := 0 < \frac{1}{2}\,v2 - \frac{\sqrt{v2\,ku\,v1}}{ku} \quad (42)$$

175. > *e31* := -sqrt(*ku*)*simplify*(*op*(2, *rhs*(*e30*)), *symbolic*) < *op*(1, *rhs*(*e30*))
·sqrt(*ku*)

$$e31 := \sqrt{v2}\,\sqrt{v1} < \frac{1}{2}\,v2\,\sqrt{ku} \quad (43)$$

176. > $e32 := lhs(e31)^2 < rhs(e31)^2$

$$e32 := v2\,v1 < \frac{1}{4}\,v2^2\,ku \quad (44)$$

177. > *solve*(*e32*, *ku*) assuming *v2* > 0

$$\left\{\frac{4\,v1}{v2} < ku\right\} \quad (45)$$

178. > ############### GO TO TOP ###############

> ##*Evaluate Solution of 2D problem*

179. > $eqV2D := -v1 + v2 \cdot U \cdot \left(1 - \frac{V}{kv}\right) = 0$

180. > $eqU2D := -u1 + u2 \cdot V \left(1 - \frac{U}{ku}\right) = 0$

$$eqV2D := -v1 + v2\,U\left(1 - \frac{V}{kv}\right) = 0$$

$$eqU2D := -u1 + u2\,V\left(1 - \frac{U}{ku}\right) = 0 \quad (46)$$

> *allvalues*(*solve*({*eqU2D*, *eqV2D*}, {*V*, *U*}))

$$\Big\{U = \Big(2\,ku^2\,\Big(\frac{1}{2}\,\frac{1}{ku\,v2}\big(ku\,kv\,u2\,v2 + ku\,u1\,v2 - kv\,u2\,v1 \quad (47)$$

$$+ \big(ku^2\,kv^2\,u2^2\,v2^2 - 2\,ku^2\,kv\,u1\,u2\,v2^2 - 2\,ku\,kv^2\,u2^2\,v1\,v2 + ku^2\,u1^2\,v2^2$$

$$- 2\,ku\,kv\,u1\,u2\,v1\,v2 + kv^2\,u2^2\,v1^2\big)^{1/2}\big) - u1\Big)\,v2\Big) \Big/ \Big(ku\,kv\,u2\,v2 + ku\,u1\,v2 - kv\,u2\,v1$$

$$+ \big(ku^2\,kv^2\,u2^2\,v2^2 - 2\,ku^2\,kv\,u1\,u2\,v2^2 - 2\,ku\,kv^2\,u2^2\,v1\,v2 + ku^2\,u1^2\,v2^2$$

$$-2\,ku\,kv\,u1\,u2\,v1\,v2+kv^{2}\,u2^{2}\,v1^{2}\big)^{1/2}\Big),\ V=\frac{1}{2}\,\frac{1}{ku\,u2\,v2}\Big(ku\,kv\,u2\,v2+ku\,u1\,v2$$

$$-kv\,u2\,v1$$

$$+\big(ku^{2}\,kv^{2}\,u2^{2}\,v2^{2}-2\,ku^{2}\,kv\,u1\,u2\,v2^{2}-2\,ku\,kv^{2}\,u2^{2}\,v1\,v2+ku^{2}\,u1^{2}\,v2^{2}$$

$$-2\,ku\,kv\,u1\,u2\,v1\,v2+kv^{2}\,u2^{2}\,v1^{2}\big)^{1/2}\Big)\Big\},\ \Big\{U=-\Big(2\,ku^{2}\,\Big(-\frac{1}{2}\,\frac{1}{ku\,v2}\Big($$

$$-ku\,kv\,u2\,v2-ku\,u1\,v2+kv\,u2\,v1$$

$$+\big(ku^{2}\,kv^{2}\,u2^{2}\,v2^{2}-2\,ku^{2}\,kv\,u1\,u2\,v2^{2}-2\,ku\,kv^{2}\,u2^{2}\,v1\,v2+ku^{2}\,u1^{2}\,v2^{2}$$

$$-2\,ku\,kv\,u1\,u2\,v1\,v2+kv^{2}\,u2^{2}\,v1^{2}\big)^{1/2}\Big)-u1\Big)\,v2\Big)\Big/\Big(-ku\,kv\,u2\,v2-ku\,u1\,v2+kv\,u2\,v1$$

$$+\big(ku^{2}\,kv^{2}\,u2^{2}\,v2^{2}-2\,ku^{2}\,kv\,u1\,u2\,v2^{2}-2\,ku\,kv^{2}\,u2^{2}\,v1\,v2+ku^{2}\,u1^{2}\,v2^{2}$$

$$-2\,ku\,kv\,u1\,u2\,v1\,v2+kv^{2}\,u2^{2}\,v1^{2}\big)^{1/2}\Big),\ V=-\frac{1}{2}\,\frac{1}{ku\,u2\,v2}\Big(-ku\,kv\,u2\,v2-ku\,u1\,v2$$

$$+kv\,u2\,v1$$

$$+\big(ku^{2}\,kv^{2}\,u2^{2}\,v2^{2}-2\,ku^{2}\,kv\,u1\,u2\,v2^{2}-2\,ku\,kv^{2}\,u2^{2}\,v1\,v2+ku^{2}\,u1^{2}\,v2^{2}$$

$$-2\,ku\,kv\,u1\,u2\,v1\,v2+kv^{2}\,u2^{2}\,v1^{2}\big)^{1/2}\Big)\Big\}$$

>